

**Mathematics and Statistics**

Date : 15/01/2022

Time: 3 h 30 m

Online Preliminary Exam

Marks: 80

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Instructions:

- i) All questions are compulsory.
  - ii) There are 6 questions divided into two sections.
  - iii) Write answers of Section – I and Section – II in the same answer book.
  - iv) Use of logarithmic tables is allowed. Use of calculator is not allowed.
  - v) For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.
  - vi) Start answer to each questions on a new page.
  - vii) For each multiple-choice type of question, it is mandatory to write the correct answer along with its alphabet. e.g. (a)..... / (b)..... /(c)..... /(d)..... No mark(s) shall be given if ONLY the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.
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**Section – I**

Q. 1. A) Select and write the correct answer of the following multiple-choice type of questions (1 mark each). (6)[12]

i) Which of the following is an open sentence ?

- a) X is a natural Number.
- b) Give me a glass of water.
- c) Wish you best of luck.
- d) I like tea

ii) If A & B square matrices of order n x n such that  $A^2 - B^2 = (A - B)(A + B)$  then

- a)  $AB = BA$
- b) either A or B is zero matrix
- c) either of A and B is an identity matrix
- d)  $A = B$

iii) If elasticity of demand  $\eta = 1$  then demand is

- a) Constant
- b) Relatively Inelastic
- c) unitary elastic
- d) elastic

iv)  $\int (x + \frac{1}{x})^3 dx =$  \_\_\_\_\_

- a)  $\frac{1}{4}(x + \frac{1}{x})^4 + c$
- b)  $\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x - \frac{1}{2x^2} + c$

c)  $\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x + \frac{1}{x^2} + c$       d)  $(x - x^{-1})^3 + c$

v)  $\int_0^2 e^x dx = \underline{\hspace{2cm}}$

- a)  $e - 1$       b)  $1 - e$       c)  $1 - e^2$       d)  $e^2 - 1$

vi) The integrating factor of  $\frac{dy}{dx} + y = e^{-x}$  is  $\underline{\hspace{2cm}}$

- a)  $x$       b)  $e^x$       c)  $-x$       d)  $e^{-x}$

B) State whether the following statements are true or false. (1 mark each) (3)

i) For  $\int \frac{(x-1)}{(x+1)^3} e^x dx = e^x f(x) dx + c$  then  $f(x) = (x + 1)^2$

ii) The area of the portion lying above the x-axis is positive.

iii) The order of highest derivative occurring in the differential equation is called degree of the differential equation.

C) Fill in the following blanks.(1 mark each). (3)

i) If  $f(x) = \frac{7}{x} - 3, x \in R, x \neq 0$  then  $f''(x) = \underline{\hspace{2cm}}$

ii) If  $f'(x) = \frac{1}{x} + x$  and  $f(1) = \frac{5}{2}$  then  $f(x) = \log x + \frac{x^2}{2} + \underline{\hspace{1cm}}$

iii) Order and degree of differential equation are always  $\underline{\hspace{2cm}}$  integers.

Q2)A) Attempt any two of the following (3 marks each) (6) [14]

i) a) Express following statement in the symbolic form and write its truth value

“ A quadratic equation has two distinct roots or 6 has three prime factors”

b) Write negation of given statement “ If 2 is prime, then 3 is odd”

ii) Find the matrix X such that  $3A - 4B + 5X = C$  where

$$A = \begin{pmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{pmatrix}, B = \begin{pmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{pmatrix}, C = \begin{pmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{pmatrix}$$

iii) Find the area of the region bounded by the curve,  $2y + x = 8$ , the X axis and  $x=2, x=4$

B) Attempt any two of the following (4 marks each) (8)

i) If the population of a town increases at a rate proportional to the population at that time. If the population increases from 40000 to 60000 in 40 years. What will be the population in another 20 years? (Given  $\sqrt{\frac{3}{2}} = 1.2247$ )

ii) If  $x^2 + 6xy + y^2 = 10$  then show that  $\frac{d^2y}{dx^2} = \frac{80}{(3x+y)^3}$

iii) For manufacturing  $x$  units, labour cost is  $150 - 54x$  and processing cost is  $x^2$ . Price of each unit is  $P = 10800 - 4x^2$ . Find the values of  $x$  for which

a) total cost is increasing    b) Revenue is increasing

Q3) A) Attempt any two of the following (3 marks each) (6) [14]

i) Find  $\frac{dy}{dx}$  if  $x = 2at^2$ ,  $y = at^4$

ii) Solve the differential equation  $y^3 - \frac{dy}{dx} = x \frac{dy}{dx}$

iii) Find the equation of the tangent and normal to the curve  $2x^2 + 3y^2 = 5$  at point (1,1).

B) Attempt any one of the following. (4)

i) Find the matrix  $X$ , if  $AX = B$  where  $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

ii) Evaluate  $\int \frac{x^2+x-1}{x^2+x-6} dx$

C) Attempt any one of the following. (Activity) (4)

i) Evaluate definite integral  $\int_1^2 \frac{3x}{(9x^2-1)} dx$

Solution :- Let  $I = \int_1^2 \frac{3x}{(9x^2-1)} dx$

$$= \square \int_1^2 \frac{x}{(9x^2-1)} dx$$

put  $9x^2 - \square = t$

differentiate,

$$\square dx = dt$$

Therefore,  $x dx = \frac{dt}{\square}$

when  $x = 1$  implies  $t = \square$

$x = 2$  implies  $t = 35$

Therefore,  $I = 3 \int_8^{35} \frac{1}{\square} \frac{dt}{18}$

$$= \frac{1}{6} \int_8^{35} \frac{dt}{\square}$$

$$= \frac{1}{6} [\log |t|]_8^{35}$$

$$= \frac{1}{6} [\log 35 - \log 8]$$

$$= \frac{1}{6} \left[ \log \frac{\square}{\square} \right]$$

ii) Using truth table verify that  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Solution:- (Fill in the blanks)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T		T	T
T	T	F	F		T	T	
T		T		T	T		T
	F	F	F	T	T	T	T
F	T	T	T			T	
F		F		F	T	F	F
	F		F	F	F	T	
F	F	F	F	F		F	F
1	2	3	4	5	6	7	8

From column 5 & 8,  $p \vee (q \wedge r) \equiv \square$

### Section II

Q.4 A) select and write the correct answer of the following multiple-choice type questions (1mark each) [6marks] (12)

1)The sum due is also called as \_\_\_\_\_.

a) face value      b) present value   c) cash value   d) true discount

2) The Assignment problem is said to be balanced if it is a\_\_\_\_\_.

a) square matrix b) Rectangular matrix c) Unit matrix d) triangular matrix .

3)The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base year  $y = 700+50x$  (in 1000 RS ) .The estimated sales for 1984(in 1000RS) is

a) RS 500 b) RS 600 c) RS 700 d) RS 750

4) Insurance companies collect a fixed amount from their customers at a fixed interval of time .this amount is called

a) Instalment b) Premium c) EMI d) contribution .

5) If  $x \sim (60, \frac{1}{30})$  then  $E(X) =$  \_\_\_\_\_.

a)2 b)5 c)4 d)3.

6) Quantity index Number by simple Aggregate Method is given by .

a)  $\sum \frac{q_1}{q_0} X 100$  b)  $\sum \frac{q_0}{q_1} X 100$  c)  $\frac{\sum q_1}{\sum q_0} X 100$  d)  $\frac{\sum q_0}{\sum q_1} X 100$

Q.4 B) State whether the following statements are true or false . [3 marks]

1) If  $b_{xy} < 0$  and  $b_{yx} < 0$  then r is positive .

2) Cyclical variation can occur several times in a year .

3) Dorbish –Bowley’s price Index number is  $P_{01}(D - B) = \frac{P_{01(L)} + P_{01(P)}}{2}$

Q.4C) Fill in the following blanks [3marks]

1) \_\_\_\_\_component of time series is indicated by periodic variation year after year .

2) If buyer is allowed both trade and cash discount , \_\_\_\_\_discount is first calculated on list price .

3) price Index number by Weighted Aggregate method is given by \_\_\_\_\_.

Q.5A) Attempt any two of the following. [6marks] (14)

1) Maximize  $Z=10x+25y$  subject to  $0 \leq x \leq 3$  ,  $0 \leq y \leq 3$  , $x +y \leq 5$  also Find maximum value of z .

2) For a certain bivariate data the following information is available

	X	Y
Mean	13	17
S.D	3	2

Correlation coefficient between X and Y is 0.6 estimate x when y=15.

3) Find the amount of an ordinary annuity if a payment of RS.500 is made at the end of every quarter for 5 years . at the rate of 12% per annum compounded quarterly

$$[(1.03)^{20} = 1.8061].$$

Q.5B) Attempt any two of the following .

[8marks]

1) A publisher produces 5 books on mathematics .the books have to go through composing ,printing and binding done by three machines P,Q,R .the time schedule for the entire task in proper unit is as

Books	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Determine the optimum time required to finish the entire task .

2) A bill drawn on the on 5<sup>th</sup> June for 6 months was discounted at the rate of 5%p.a. on 19<sup>th</sup> October .if the cash value of the bill is Rs.43,500 ,find Face value of the bill.

3) Suppose error involved in making a certain measurement is a continuous r.v. X with p.d.f

$$f(x) = k(4 - x^2) \quad \text{for } -2 \leq x \leq 2$$

$$= 0 \quad \text{otherwise}$$

Compute P ( $x < -0.5$  or  $x > 0.5$ ).

Q.6A) Attempt any two of the following.

[6marks] (14)

i) In a multiple-choice test with three possible answers for each of the five questions what is the probability of a candidate getting four or more correct answers by random choice .

ii) Find the expected value and variance of X using the following p. m. f

X	-2	-1	0	1	2
P(x)	0.2	0.3	0.1	0.15	0.25

iii) In a cattle breeding firm ,it is prescribed that the food ration for one animal must contain 14,22 and 1 unit of nutrients A,B and C respectively two different types of fodder are available each unit weight of these two contains the following amount of these three nutrients.

	Nutrients	Fodder1	Fodder2
Fodder			
Nutrients A		2	1
Nutrients B		2	3
Nutrients C		1	1

The cost of fodder 1 is Rs.3 per unit and that of fodder 2 is Rs. 2 per unit .Formulate the LPP to minimize the cost.

Q6B) Attempt any one of the following [4marks]

i) The following data gives the production of bleaching powder (in 000'tons) for the year 1962 to 1972.

Year	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
Production	0	0	1	1	4	2	4	9	7	10	8

Fit a trend line by the method of least square also obtain the trend value for the year 1975.

ii) The data obtained on X the length of time in weeks that a promotional project has been in progress at a small business and Y the percentage increase in weekly sales over period just prior to the beginning of the campaign

X	1	2	3	4	1	3	1	2	3	4	2	4
Y	10	10	18	20	11	15	12	15	17	19	13	16

Find the equation of regression line to predict the percentage increase in sales if the campaign has been in progress for 1.5 weeks .

Q.6.C)Attempt any one of the following activity [4 marks]

i) Find x ,if the cost-of-living Index is 150

Group	Food	Clothing	Fuel and lightning	House Rent	Miscellaneous
I	180	120	300	100	160
W	4	5	6	X	3

Solution -

Group	I	W	IW
Food	180	4	720
Clothing	120	5	
Fuel and Lighting	300	6	1800
House Rent	100	x	
Miscellaneous	160	3	480
Total			

Now by family budget method ,

CLI =

$$150 = \frac{100x+3600}{x+18}$$

150x +  = 100x + 3600

150x - 100x = 3600 -

50x = 900

X =

ii) For the following assignment problem minimize total man hours

Subordinates	Required hours for task			
	I	II	III	IV
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9



Solution –

Subtract the smallest element of each row from every element of that row

Subordinates	Required hours for task			
	I	II	III	IV
A	0	18	19	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Subtract the smallest element in each column from every element of that column

Subordinates	Required hours for task			
	I	II	III	IV
A		14	9	3
B	9		0	22
C	23	0		0
D	9	12	14	0

Optimum solution is shows as follows

A →  B →  C →  D →

Minimum hours required is =  hrs.

-----All THE BEST-----